



Worksheet 3 Karnaugh maps **Answers**

Task 1

- Fill in Karnaugh maps for the following expressions, showing the groupings by ringing them.

In parts (d), (e) and (f), fill in the missing row and column labels.

(a) A

(b) $\neg A \wedge C$

BC^A

	00	01	11	10
0				
1	1	1	1	1

	00	01	11	10
0		1	1	
1				

(c) $A \wedge \neg B$

(d) B

(e) $A \wedge B \wedge \neg C$

BC^A

	00	01	11	10
0				
1	1	1		

	00	01	11	10
0			1	1
1			1	1

	00	01	11	10
0				
1				1

- Fill in Karnaugh maps for the following expressions, showing the groupings. Hence simplify the expressions.

(a) $(A \wedge B) \vee (A \wedge \neg C) \vee (A \wedge \neg B) = A$

(b) $(A \wedge B \wedge C) \vee (\neg A \wedge B) \vee (A \wedge B \wedge \neg C) = B$

BC^A

	00	01	11	10
0				
1	1	1	1	1

	00	01	11	10
0			1	1
1			1	1

- Use a Karnaugh map to show that $A \vee \neg A \wedge B = A \vee B$.

(You drew a truth table to prove this in Worksheet 2, Question 4)

B^A

	0	1
0		1
1	1	1



Task 2

4. What Boolean expressions do each of the ringed squares in the Karnaugh map in Figure 1 represent?

AB
CD

	00	01	11	10
00			1	
01				
11				
10			1	

Figure 1

Row 1 column 3 = $(\neg A \wedge \neg B \wedge C \wedge D)$

Row 4 column 3 = $(A \wedge \neg B \wedge C \wedge D)$

Write the Boolean expression represented by the map in its simplest form.

The two ringed squares form one group which "wraps around" to represent $\neg B \wedge C \wedge D$

5. (a) Ring the two groups in Figure 2. What Boolean expression does this Karnaugh map represent? $(\neg B \wedge C \wedge D) \vee (A \wedge B \wedge D)$

AB
CD

	00	01	11	10
00			1	
01				
11		1	1	
10			1	

Figure 2



(b) Complete the Karnaugh map in Figure 3 to represent the expression:

$$(A \wedge B \wedge C \wedge \neg D) \vee (\neg A \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge C \wedge \neg D)$$

Draw the resulting group(s) and hence simplify the expression.

Simplified expression: $C \wedge \neg D$

AB
CD

	00	01	11	10
00				1
01				1
11				1
10				1

Figure 3

(c) Complete the Karnaugh map in Figure 4 to represent the expression:

$$(A \wedge B \wedge C) \vee (C \wedge D) \vee (A \wedge \neg C) \vee (A \wedge \neg B \wedge C \wedge \neg D)$$

Draw the groups, and hence simplify the expression.

Simplified expression: $A \vee (C \wedge D)$

AB
CD

	00	01	11	10
00			1	
01			1	
11	1	1	1	1
10	1	1	1	1

Figure 4



(d) Complete the Karnaugh map in Figure 5 to represent the expression:

$$(\neg A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (A \wedge \neg B \wedge C \wedge \neg D)$$

Draw the group(s), and hence simplify the expression.

Simplified expression: $\neg B \wedge \neg D$

Note: When the corners are filled in, they all wrap around to make one group similar to a folded piece of paper.

AB
CD

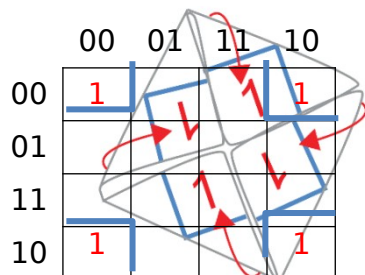


Figure 5

(e) Complete the Karnaugh map in Figure 6 to represent the expression:

$$(\neg A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (A \wedge \neg B \wedge C \wedge \neg D) \vee (\neg B \wedge D)$$

Draw the group(s), and hence simplify the expression.

Simplified expression: $\neg B$

The top and bottom rows wrap round to form a single group

AB
CD

	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

Figure 6

7. How many squares in a Karnaugh map with 4 variables contain 1 when an expression containing only AND symbols has:

(i) 4 variables, e.g. $(\neg A \wedge \neg B \wedge \neg C \wedge \neg D)$? **1**

(ii) 3 variables e.g. $(A \wedge B \wedge C)$? **2**

(iii) 2 variables, e.g. $(\neg B \wedge D)$? **4**

(iv) 1 variable, e.g. A ? **8**